

Speed Math Solutions

CHILES MINI MU

December 9, 2023

1. Evaluate

$$1 + 2(1 + 2(1 + 2(1 + 2(1 + 2(1 + 2))))).$$

Solution. Note that $1 + 2 = 2^2 - 1$, and after multiplying by 2 and adding 1, this becomes $2^3 - 1$. This pattern continues, and the answer is $2^7 - 1 = \boxed{127}$. \square

2. How many zeroes are at the end of 64% of 6,250,000?

Solution. This equals $(2^6 \cdot 10^{-2}) \cdot (5^4 \cdot 10^4) = 2^2 \cdot 10^6$, so the answer is $\boxed{6}$. \square

3. What is the number of distinguishable permutations of *KINGJAMES*?

Solution. All nine letters are distinct, so the answer is $9! = \boxed{362880}$. \square

4. Let \mathcal{X} be the smallest positive integer such that there exists a polygon with an interior angle sum of $\mathcal{X}!$ degrees. What is \mathcal{X} ?

Solution. Since the interior angle sum of an n -gon is $180(n - 2)$, we need $180 \mid \mathcal{X}!$. The smallest such \mathcal{X} is $\boxed{6}$. \square

5. The fraudulent basketball player Joel Embiid is practicing shooting free throws. He has already attempted 2024 free throws and made 1600 of them. How many does he now have to make in a row to raise his success rate to 90%?

Solution. Or, we can set up the equation $\frac{1600+x}{2400+x} = \frac{9}{10}$, where x is the number of free throws he makes in a row. Solving for x gives $\boxed{2216}$. \square

6. What is the closest integer to 2% of 16% of 1% of 8% of 4% of one billion (1,000,000,000)?

Solution. This is equal to

$$(2^1 \cdot 10^{-2})(2^4 \cdot 10^{-2})(2^0 \cdot 10^{-2})(2^3 \cdot 10^{-2})(2^2 \cdot 10^{-2}) \cdot 10^9 = 2^{10} \cdot 10^{-1} = 102.4,$$

so the answer is $\boxed{102}$. \square

7. Given reals x, y satisfying $3x + 8y = 20$ and $2x + 7y = 24$, compute $x + 2y$.

Solution. Note that subtracting twice the second equation from three times the first equation gives $5x + 10y = 12$, so dividing by 5 gives $x + 2y = \boxed{\frac{12}{5}}$. \square

8. Shaoyang and Lily are on opposite sides of a field. They begin running back and forth across the field at the same time. If it takes Shaoyang and Lily 20 and 24 minutes to run the length of the field, respectively, after how many minutes will they be at opposite ends of the field again?

Solution. In order for both Shaoyang and Lily to be at the ends of the field, the answer must be divisible by $\text{lcm}(20, 24) = 120$. However, after 120 minutes, Shaoyang is on the same side of the field that he was originally on, as $120/20$ is even, while Lily is on the opposite side of the field that she started on, as $120/24$ is odd. At this point, they are on the same side of the field, so 120 doesn't work. However, after $\boxed{240}$ minutes, both Shaoyang and Lily are on the side they originally started on, as desired. \square

9. Find the number of two-digit positive integers factors of $5!$.

Solution. Since $5! = 120 = 2^3 \cdot 3 \cdot 5$, there are $(3+1)(1+1)(1+1) = 16$ total factors of 120. There are 8 factors of 120 that aren't two digits: 1, 2, 3, 4, 5, 6, 8, and 120. Thus, there are $16 - 8 = \boxed{8}$ two-digit factors of $5!$. \square

10. What is the units digit of $2 \cdot 202^4$?

Solution. Note that $2 \cdot 202^4 \equiv 2 \cdot 2^4 \equiv 2 \cdot 16 \equiv 32 \equiv 2 \pmod{10}$, so the answer is $\boxed{2}$. \square

11. Compute

$$42 + \frac{43}{42 + \frac{43}{42 + \dots}}$$

Solution. Let x be the value of the expression. Then $x = 42 + 43/x$, so $x^2 - 42x - 43 = 0$, and taking the positive root gives $x = \boxed{43}$. \square

12. What is 101101000101_2 in base 4? Do not include the base 4 subscript in your answer.

Solution. Separating the base 2 number into groups of two digits, we get that

$$101101000101_2 = \boxed{231011}_4$$

as desired. \square

13. There are five little monkeys jumping on the bed. At the end of every minute, if there are n monkeys remaining on the bed, there is a $\frac{1}{n+1}$ chance that exactly one monkey will fall off the bed, and $\frac{n}{n+1}$ chance that no monkeys fall off. What is the probability that all the monkeys will fall off the bed after 5 minutes?

Solution. Every minute, a monkey must fall off, so the answer is $\frac{1}{6} \cdot \frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} = \boxed{\frac{1}{720}}$. \square

14. What is the area of the quadrilateral with vertices $(20, 24)$, $(-20, 24)$, $(2, 2)$, and $(20, 2)$?

Solution. Graphing, we see that this is a trapezoid with base lengths 18 and 40 and height 22. The area is $\frac{22(18+40)}{2} = 11 \cdot 58 = \boxed{638}$. \square

15. Linsey takes a nap at 3 : 17 and sleeps for 4 hours and 44 minutes. What time does she wake up?

Solution. Four hours after 3 : 17 is 7 : 17, and 44 minutes after that is $\boxed{8 : 01}$. \square

16. What is

$$\sqrt{1} + \sqrt{3} + \sqrt{9} + \sqrt{27} + \sqrt{81} + \sqrt{243} + \sqrt{729}$$

expressed in simplest radical form?

Solution. This is equal to

$$1 + \sqrt{3} + 3 + 3\sqrt{3} + 9 + 9\sqrt{3} + 27 = \boxed{40 + 13\sqrt{3}},$$

as desired. □

17. How many triangles can be formed from the vertices of a regular hexagon?

Solution. The answer is $\binom{6}{3} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = \boxed{20}$. □

18. Aaron is playing with three rhesus monkeys and four barbary apes. How many ways can all 8 of them stand in a line given that animals of the same species are indistinguishable?

Solution. The answer is just $\frac{8!}{4! \cdot 3! \cdot 1!} = \boxed{280}$. □

19. Let $a \star b = ab - a - b + 100$. Compute $1 \star (2 \star (3 \star (4 \star 5)))$.

Solution. Note that $1 \star b = b - 1 - b + 100 = 99$, regardless of the value of b , so the answer is $\boxed{99}$. □

20. Using the notation from the previous problem, evaluate $p \star q$, where p and q are the roots of $x^2 - 44x + 480$.

Solution. From Vieta, $p + q = 44$ and $pq = 480$, so $p \star q = 480 - 44 + 100 = \boxed{536}$. □

21. Nima has a 20 liter solution that is 40% acid and 60% water. How many liters of acid does he need to add to his solution to make it 60% acid?

Solution. The amount of water in the solution doesn't change at all, and it stays at $0.6 \cdot 20 = 12$ liters. For this to be 40% of the solution, the total amount of solution must be $2.5 \cdot 12 = 30$ liters, so Nima needs to pour $30 - 20 = \boxed{10}$ liters of acid. □

22. Yimo is deciding on what to wear for the day. He has 6 shirts, 5 pairs of pants, and 10 pairs of socks to choose from. Given that his outfit consists of one shirt, one pair of pants, and one pair of socks, how many different outfits can he choose?

Solution. This is just $6 \cdot 5 \cdot 10 = \boxed{300}$. □

23. Evaluate the expression

$$\frac{3 + \frac{3}{4}}{4 + \frac{4}{3}}.$$

Solution. The numerator is $\frac{15}{4}$ and the denominator is $\frac{16}{3}$, so the answer is $\frac{15}{4} \cdot \frac{3}{16} = \boxed{\frac{45}{64}}$. □

24. David can take a picture every 40 seconds. If Nonoko joins David, they can take a picture every 15 seconds. If both David and Nonoko take pictures at a constant rate, how many seconds does it take Nonoko to take a picture if she works alone?

Solution. Let r be the rate the Nonoko takes pictures at in pictures per second. Then

$$r + \frac{1}{40} = \frac{1}{15} \implies \frac{1}{r} = \frac{25}{40 \cdot 15} = \frac{1}{24},$$

so the answer is $\boxed{24}$. □

25. Compute $(6 \times 8 - 4/2)(2^2 + 4 \times 6 - 8)$.

Solution. From Order of Operations, this equals

$$(48 - 2)(4 + 24 - 8) = (46)(20) = \boxed{920},$$

as desired. □